Antileakage Fourier transform for seismic data regularization

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ABSTRACT
Seismic data regularization, which spatially transforms irregularly sampled acquired data to regularly sampled data, is a long-standing problem in seismic data processing. Data regularization can be implemented using Fourier theory by using a method that estimates the spatial frequency content on an irregularly sampled grid. The data can then be reconstructed on any desired grid. Difficulties arise from the nonorthogonality of the global Fourier basis functions on an irregular grid, which results in the problem of “spectral leakage”: energy from one Fourier coefficient leaks onto others.

We investigate the nonorthogonality of the Fourier basis on an irregularly sampled grid and propose a technique called “antileakage Fourier transform” to overcome the spectral leakage. In the antileakage Fourier transform, we first solve for the most energetic Fourier coefficient, assuming that it causes the most severe leakage. To attenuate all aliases and the leakage of this component onto other Fourier coefficients, the data component corresponding to this most energetic Fourier coefficient is subtracted from the original input on the irregular grid. We then use this new input to solve for the next Fourier coefficient, repeating the procedure until all Fourier coefficients are estimated. This procedure is equivalent to “reorthogonalizing” the global Fourier basis on an irregularly sampled grid.

We demonstrate the robustness and effectiveness of this technique with successful applications to both synthetic and real data examples.

INTRODUCTION
Marine seismic data are usually irregularly and sparsely sampled along the spatial directions for several reasons, including cable feathering, editing of bad traces, and economics. However, regularly sampled data are required for applications of 3D surface-related multiple elimination (SRME) (Verschuur et al., 1992; Weglein et al., 1997) and 3D wave equation migration (WEM) (Claerbout, 1971; Zhang et al., 2003). The best way to obtain 3D regularly sampled data is to acquire more data, with more redundancy in the crossline direction and with a wider azimuth range, but this is expensive and difficult to achieve because of current marine-acquisition technology. Therefore, data regularization becomes an important processing step that needs to be addressed.

Three-dimensional seismic data regularization requires generating seismic traces at locations where the actual experiment at the source and receiver position has not taken place. In other words, one has to interpolate or extrapolate seismic traces from the acquired data on the irregular grid to the regular grid. We follow the definition of interpolation given by Thévenaz et al. (2000): model-based recovery of continuous data from discrete data with a known range of abscissa. This definition allows for a clear distinction between interpolation and extrapolation. Thévenaz et al. (2000) gave the three most important requirements for a successful interpolation:

1) The underlying data are defined continuously.
2) Given data samples, it is possible to compute a data value of the underlying continuous function at any abscissa.
3) The evaluation of the underlying continuous function at the sampling points yields the same values as the given data.

We assume marine seismic data roughly meet these requirements, and we strive to reconstruct by interpolation the continuous reflection or diffraction information carried by irregularly sampled seismic data. Actually, the dominant frequency of seismic wavelets is relatively low, and seismic data are usually oversampled along the time direction. So the seismic data in the time direction are continuous and have higher resolution. On the other hand, the multifold data possess redundant information along the lateral space directions. All these make seismic data reconstruction possible.
Many seismic data regularization techniques have been proposed in the past few years. The simplest data regularization is the binning method; that is, “bin” the data and disregard the true data acquisition locations in the bin. This binning may cause artifacts and contaminate the final migrated images. Binning methods with partial normal moveout (NMO) to correct for differential movement of shot and receiver locations yield a better result. However, they still suffer from the oversimplified assumptions of NMO.

Another seismic data regularization approach is built on the integral of continuation operators (normally, Kirchhoff type) (Bagamini and Spagnolini, 1996; Canning and Gardner, 1996; Bleistein and Jaramillo, 2000; Chemingui and Biondi, 2002; Stolt, 2002). These algorithms depend on the velocity model and generally give poor results at near offsets because of limited integration apertures. They also suffer from irregularities in the input geometry. Incorporating inversion theory offers better results but at an increased expense of computational cost.

Seismic data regularization can also be obtained by finding a convolution filter that predicts the data in such a way that the error is white noise. Generally, the filter is calculated by inversion methods. The prediction error filter (PEF) is the most commonly used such filter (Abma and Claerbout, 1995). The main advantage of the PEF is that it can handle aliased linear events (Spitz, 1991; Mazzucchelli et al., 1998) and nonstationary events (Crawley, 2000). This method can fill in gaps in a regularly sampled data set and can be used for reconstruction. The main drawback of this method is its use of local linear events approximation.

Fourier methods (Duijndam et al., 1999; Hindriks and Duijndam, 2000; Schonewille, 2000) provide another way of doing seismic data regularization. These methods are normally tied to global, discrete transforms (e.g., Fourier transform or Radon transform). These transforms are very efficient on regularly sampled grids, but when they are applied to an irregularly sampled data set, the transformed data may be altered by noise due to the spectral leakage among different frequency components in the Fourier reconstruction-based regularization. Spectral leakage is a well-known problem in experimental physics (Press et al., 1993; Zwartjes and Gisolf, 2002). In astronomy, for example, it has been addressed by a method consisting of successively estimating the strongest spectral component from the data (Gray and Desikachary, 1973; Lomb, 1976). In the present paper, we address seismic trace interpolation using this kind of approach.

We first briefly review Shannon’s sampling theorem and the nonuniform Fourier transform. We show a simple example illustrating the nonorthogonality property and the associated spectral-leakage phenomenon. Then, we propose a practical solution, called the antileakage Fourier transform (ALFT) to overcome the problem of spectral leakage caused by the irregular sampling. Our approach is to reorthogonalize the global Fourier basis on its irregularly sampled grid, resulting in an accurate spectrum of Fourier coefficients for irregularly sampled data. In the point of view of reorthogonalization, our method is similar to the method of Gray and Desikachary (1973), although we do not use irregular sampling wavelet deconvolution. Instead, we estimate the data weighting described in Zwartjes and Gisolf (2002). We next demonstrate the efficiency of our approach with a test on synthetic data. Finally, an application to a real seismic-trace interpolation problem reveals the robustness and effectiveness of the proposed algorithm.

**SAMPLING THEORY AND FOURIER BASIS ON REGULAR GRID**

Shannon’s (1949) milestone paper set the foundation of information theory (Bracewell, 1965). For converting an analog signal into a sequence of numbers, Shannon stated the classical sampling theorem in the following terms.

If a function \( f(x) \) contains no frequencies higher than \( \omega_{\text{max}} \) (in radians per second), it is completely determined by giving its values at a series of points spaced \( T = \pi/\omega_{\text{max}} \) seconds apart.

The reconstruction formula also was given by Shannon:

\[
 f(x) = \sum_{k \in \mathbb{Z}} f_k \varphi_k = \sum_{k \in \mathbb{Z}} f(kT) \text{sinc} \left( \frac{x}{T} - k \right), \quad (1)
\]

where \( \mathbb{Z} \) denotes the set of integers, the basis function \( \varphi_k(x) \) is the sinc function \( \varphi_k(x) = \text{sinc}(x/T - k) \) with \( \text{sinc}(x) = \sin(\pi x)/\pi x \). Equation 1 is exact if \( f(x) \) is band-limited to \( \omega_{\text{max}} < \pi/T; \) this upper limit is the Nyquist frequency, a term that was coined by Shannon in recognition of Nyquist’s important contributions in communication theory. The basis functions \( \varphi_k \) are orthogonal as long as \( k, \ell \) are integers:

\[
 \langle \varphi_k, \varphi_\ell \rangle = \int_{-\infty}^{+\infty} \text{sinc} \left( \frac{x}{T} - k \right) \text{sinc} \left( \frac{x}{T} - \ell \right) dx = \text{sinc}(k - \ell) = \delta_{k\ell}, \quad (2)
\]

where \( \delta_{k\ell} \) is the Kronecker delta function (\( \delta_{k\ell} = 1 \) if \( k = \ell \) and \( \delta_{k\ell} = 0 \) otherwise). The sinc functions also satisfy the unity condition; i.e., for any real number \( x \in \mathbb{R} \),

\[
 \sum_{k \in \mathbb{Z}} \text{sinc}(x + k) = 1. \quad (3)
\]

The orthogonality property (equation 2) greatly simplifies the implementation of data reconstruction. It assures that on a regular grid \( (k, \ell \in \mathbb{Z}) \), the sinc function has the weight 1 on the original data position and zero weight on all other integer locations (Unser, 2000).

On an arbitrary irregular grid, the orthogonality condition (equation 2) and unity condition (equation 3) do not hold. Furthermore, the concept of Nyquist frequency does not explicitly exist. For practical implementation, one needs to cut off at some maximum frequency. This will lead to a sinc function centered at each point of the original irregularly sampled grid. The uniform sinc function can take nonzero values at the location of the other samples, which might not be integers. Thus, using the uniform sinc function for irregularly sampled data reconstruction will result in an incorrect interpolation because it violates both the orthogonality condition and the unity condition.

It is easier to rebuild the unity condition on an irregular grid. In engineering, attention was focused on this unity condition, which does not hold on an irregular grid; i.e., for a general
where $\text{Fourier integral over a unit integral range:}$

\[ \sum_{p \in N_p} \varphi(x + p) \neq 1. \tag{4} \]

Here, $N_p$ denotes the index set of irregular data samples. In equation 4, the reconstruction has nonnormalized weights. Usually, one can normalize the unity condition with some weighting $w$ applied to the data. Denoting $\varphi_p = \varphi(x/T - p)$, then the normalized reconstruction becomes

\[ f(x) = \frac{\sum_{p \in N_p} f_p w \varphi_p}{\sum_{p \in N_p} w \varphi_p}. \tag{5} \]

If one introduces a B-spline interpolant for $\varphi_p$, equation 5 becomes the basic formula for nonuniform rational B-spline (NURBS), first proposed by Versprille (1975), and widely applied in computer graphics for representing free-form curves and surfaces.

A reconstruction method based on equation 5 satisfies the unity condition and can rebuild a smooth free surface. However, it does not meet the orthogonality condition; therefore, the reconstructed data do not fit the original measurements on the irregular grid.

In this paper, we work on interpolation methods on irregular grids; thus we have to pay attention to the orthogonality condition, which is considered to be the most important condition in data reconstruction (Unser, 2000).

**THE NONUNIFORM FOURIER TRANSFORM AND ITS INVERSE**

The fast Fourier transform (FFT) algorithm (Cooley and Tukey, 1965; Press et al., 1993, section 13.9) has significant limitations in many applications because of its requirement of sampling on an equally spaced grid. Fourier transformation of irregularly sampled data may be realized by direct evaluation of trigonometric sums:

\[ \hat{f}(k) = \sum_{\ell \in N_p} f(x_\ell) e^{-2\pi i k x_\ell}, \quad k = 0, \pm 1, \ldots, \pm N. \tag{6} \]

This is more costly than the FFT requiring $O(N \times N_p)$ operations. The nonuniform FFT (NFFT) was developed for fast computation of equation 6 (Dutt and Rokhlin, 1993; Beylkin, 1995; Duijndam and Schonewille, 1999). Their methods are fast and accurate for computing the summation in equation 6 (reduce the operation to $O(N \log N)$). However, the Fourier coefficients in equation 6 are not adequate for data interpolation since equation 6 is not even the discretization of the Fourier integral over a unit integral range:

\[ \hat{f}(k) = \int f(x) e^{-2\pi i k x} dx \approx \sum_{\ell \in N_p} f(x_\ell) \Delta x_\ell e^{-2\pi i k x_\ell}, \tag{7} \]

where $\Delta x_\ell$ is the data weight in the summation. With variable $\Delta x_\ell$, this sum provides a discrete summation with a better approximation of the Fourier integral. In one dimension, $\Delta x_\ell$ is easy to estimate because it is the interval of adjacent samples, but it is more difficult to estimate in higher dimensions. In any case, equation 7 still cannot give accurate Fourier coefficients whenever there exist some large range of values $\Delta x_\ell$ (i.e., when the data sampling is highly irregular).

The nonorthogonality problem may be partially solved by inversion theory (Sacchi and Ulrych, 1995; Liu and Tang, 1998; Zwartjes and Hindriks, 2001; Wang, 2003). In Sacchi and Ulrych’s (1995) method, the Fourier coefficients were obtained by the minimization of the cost function:

\[ C = \sum_{\ell \in N_p} \left\| \frac{1}{N_p} \sum_{k=1}^{N_k} \hat{f}(k)e^{2\pi i k x_\ell} - f(x_\ell) \right\|^G + \sigma \sum_{k=1}^{N_k} \| \hat{f}(k) \|^C, \tag{8} \]

where $\| \cdot \|^G$ and $\| \cdot \|^C$ denote the norm of the Gaussian distribution and norm of the Cauchy distribution, respectively. The reconstruction based upon equation 8 does not satisfy one of the desired conditions for the interpolator: it does not honor the data. The reason for this is that inversion allows misfits between the reconstructed continuous data and the original measurements, although the estimated Fourier coefficients can normally represent the data at the original irregular grid with accuracy. Practically, for noisy data, interpolating seismic traces to a regular grid is still difficult, because it is hard to obtain a proper damping factor $\sigma$ in a real data implementation. Too small a value of $\sigma$ will make the inversion unstable, and too large a value will degrade the result. Furthermore, the assumption of sparseness is also questionable.

**IRREGULARITY AND NONORTHOGONALITY**

The relation between irregularities in data sampling and the nonorthogonality of Fourier basis on the irregular grid identifies the fundamental problem of data regularization. A simple synthetic test will help us to understand this phenomenon. Figure 1 depicts a simple sinusoidal signal sampled on an irregular grid. The positions of sampling are generated by a random function:

\[ x_\ell = 128 \times 1l, \quad \ell = 1, 2, \ldots, 128, \tag{9} \]

where $1l$ are random numbers [created by ran2.f in Press et al. (1993, section 7.1)]. Re-sorting $x_\ell$ in ascending order, we generate the synthetic data with the function

\[ f(x_\ell) = \sin \left( \frac{x_\ell}{5} \right). \tag{10} \]

This sinusoid lasts about four periods on the abscissa $(0, 128)$ (see Figure 1). It is a low-frequency signal on a highly irregular grid. In this synthetic test, the largest interval $\Delta x_{\text{max}}$ between samples is 6.1511; it is obvious that the signal is far from being aliased even at a sample rate of $\Delta x_{\text{max}}$. Thus, it is not difficult to obtain the signal spectrum with fair accuracy just by taking the Fourier summation 6 or discrete Fourier integral 7.

After applying the Fourier summation of equation 6, we show the spectrum in Figure 2. The actual spectrum is nonzero but it is more costly than the FFT requiring $O(N \times N_p)$ operations. The nonuniform FFT (NFFT) was developed for fast computation of equation 6 (Dutt and Rokhlin, 1993; Beylkin, 1995; Duijndam and Schonewille, 1999). Their methods are fast and accurate for computing the summation in equation 6 (reduce the operation to $O(N \log N)$). However, the Fourier coefficients in equation 6 are not adequate for data interpolation since equation 6 is not even the discretization of the Fourier integral over a unit integral range:

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where $\Delta x_\ell$ is the data weight in the summation. With variable $\Delta x_\ell$, this sum provides a discrete summation with a better approximation of the Fourier integral. In one dimension, $\Delta x_\ell$ is easy to estimate because it is the interval of adjacent samples, but it is more difficult to estimate in higher dimensions. In any case, equation 7 still cannot give accurate Fourier coefficients whenever there exist some large range of values $\Delta x_\ell$ (i.e., when the data sampling is highly irregular).
everywhere, spreading over almost all frequencies in the range of the Fourier summations. This phenomenon is well known as spectral/frequency leakage. The energy leakage is caused by the irregularities of sampling and boundary effects. It is the energy of the signal (spike spectrum) in the data that leaks to all the other frequencies when the summations are performed on the irregularly sampled grid. The amplitudes of the leakage spectrum are large, mostly around 15–20% of the signal spectrum amplitude, and the maximum leakage is more than 30% of the signal spectrum amplitude. From the spectrum given in Figure 2, it is possible to recover the original irregularly spaced data set. However, the attempt to recover the missing data of the simple sinusoid on a regular grid fails because the correct interpolation spectrum should be a single spike in the Fourier domain. Additionally, one can see that the signal spike is very close to but not exactly at the expected location of \( k \approx 4.0743665 \). This may be because the Fourier summation (equation 6) on an irregular grid cannot give the exact Fourier coefficients, although the accuracy is good enough for practical applications.

**ANTILLEAKAGE FOURIER TRANSFORM**

To attenuate the leakage of Fourier coefficients, we consider now the normalized Fourier summation for a single value \( k \) of the Fourier transform variable (below, we refer to \( k \) as “frequency”). The forward and inverse Fourier transforms are defined as

\[
\hat{f}(k) = \frac{1}{\Delta X} \sum_{\ell \in \mathbb{Z}} \Delta x_\ell f(x_\ell) e^{-2\pi i k x_\ell},
\]

\[
f^k(x_\ell) = \hat{f}(k) e^{2\pi i k x_\ell},
\]

where \( \Delta X \) is the summation range, \( \hat{f}(k) \) is the Fourier coefficient for frequency \( k \), and \( f^k(x_\ell) \) denotes the component of frequency \( k \) in the input data. On a regularly sampled grid, \( f^k(x_\ell) \) will only affect the estimation of the Fourier coefficient at frequency \( k \) because of the Fourier orthogonality condition. But on an irregularly sampled grid, the orthogonality condition fails, and the nonzero \( f^k(x_\ell) \) will leak to all other frequencies.

Although for standard Fourier summation the sequence of solving for each Fourier coefficient has no effect on the final results, it is crucial in our algorithm because the Fourier coefficients of larger magnitude will have more leakage than those with smaller ones. To reduce the leakage, we propose an ALFT, which works by estimating Fourier coefficients recursively, beginning with the one with the maximum energy and proceeding downward in energy (or magnitude) to the one with the minimum energy. After each step of estimation, the calculated \( \hat{f}(k) \) will be set to zero by updating the input data. Mathematically, it is equivalent to removing the component \( f^k(x_\ell) \) from the input data:

\[
f^u(x_\ell) = f(x_\ell) - f^k(x_\ell). \tag{12}
\]

Therefore, our regularization algorithm on an irregular grid can be implemented in the following steps:

1) Compute all Fourier coefficients of the input data using equation 11.
2) Select the Fourier coefficient with the maximum energy.
3) Subtract the contribution of this coefficient from the input data (equation 12).

We then use this newly subtracted input to solve for the next Fourier coefficient with the same maximum-energy criterion. We repeat the procedure until all Fourier coefficients are resolved, i.e., until all the values in the updated input tend to zero (practically, below a threshold). Based on this procedure, there is no need to taper the data boundary to mitigate wraparound effects. We assume here that the global basis functions are orthogonal (which was shown for a regular grid for Fourier basis in equation 2). Equation 12 acts as an orthogonalization mechanism for the Fourier basis on an irregular grid. In other words, after the operations in equation 12, the Fourier basis is reorthogonalized. This leads to a practical solution for minimizing the leakage effect from one frequency to another. Furthermore, the final updated input data on the irregular grid will tend to zero after all the subtraction operations. This implies that the reconstructed data from the obtained Fourier coefficients will fit the original measurements. Therefore, the proposed data regularization method meets all interpolation requirements.

On a regular grid, this subtraction is trivial because of the orthogonality condition of the Fourier basis. Therefore, in the regular sampling case, the Fourier coefficients from the ALFT are identical to those from the FFT. But because of the subtraction, our algorithm is not as efficient as FFT.

For noise-free data with a single frequency, the result is predictable. After solving for the first maximum-energy spectrum, the data residuals in equation 12 become small. As a consequence, for other frequencies, the leakage source is no longer significant. This means that subsequent operations of solving for all the other frequencies will suffer much less from the effect of frequency leakage.

Figure 3 shows the result of applying the ALFT to the data of Figure 1; it shows a single frequency with almost no leakage. With Fourier coefficients computed by ALFT, one can easily reconstruct the original signal on the regular grid with an ordinary FFT algorithm.

Normally, the ALFT works on the framework of nonaliased signal reconstruction. Similarly to the result in Gray and Desikachary (1973), our method also handles aliased events, thus making the ALFT method a practical approach to seismic data regularization.

To test the effectiveness of this algorithm on data having different frequencies and different amplitudes, we use the
following function:

\[ f(x_\ell) = 5 \sin \left( \frac{x_\ell}{5} \right) + 2.5 \sin(x_\ell + 1). \]  

(13)

The sampling locations \( x_\ell \) are the same as in Figure 1. Figure 4 shows the generated input data. Figure 5 shows the spectrum computed by equation 6. Only one signal component on this spectrum map can be distinguished as the strongest signal component. The spectrum of the weak signal is buried in the leakage spectrum. Here, the amplitude of the maximum leakage spectrum reaches 40% of that of the strongest signal. With Fourier coefficients computed by equation 6, reconstruction is difficult on a regular grid. The result of the ALFT is displayed in Figure 6. We can see the two distinct peaks in the spectrum. After the ALFT, the Fourier coefficients can be used for signal reconstruction on any regular grids.

In real data, the presence of noise may cause a problem for the stability of any algorithm. To demonstrate the robustness of our approach, we consider the following function with added random noise:

\[ f(x_\ell) = 5 \sin \left( \frac{x_\ell}{5} \right) + 2.5 \sin(x_\ell + 1) + 6(\Re_\ell - 0.5), \]  

(14)

where \( \Re_\ell \) denotes the random noise associated to sample \( \ell \). In equation 14, the amplitude of the random noise is in the range \((-3, 3)\), which is stronger than the amplitude of the second signal. The same irregularly sampled grid is used, as for the previous two experiments. The irregularly sampled data are shown in Figure 7. Compared to the data shown in Figure 4, the data in Figure 7 have more oscillations, and the maximum absolute value is about 10.

The spectrum of the discrete Fourier summation (equal to the NFFT spectrum) can be obtained by using equation 6 (see Figure 8). Random noise is added to the input data, but the strongest signal spectrum has not changed much compared to Figure 5. A possible explanation for this fact is that the noise is random. Figure 9 is the result of our ALFT, which shows that
the method is stable in the presence of random noise. Two distinguishable peaks of the signals are clearly visible in the spectrum.

**EXTENSION TO MULTIDIMENSIONS**

In higher dimensions, equation 11 in the frequency domain and the wave-number domain becomes a set of linear equations with parameters of wave-number vector $\mathbf{k}$ and the position vector $\mathbf{x}_\ell$, respectively. The Fourier transform/summation and its inverse for a single $\mathbf{k}$ becomes

$$\hat{f}(\mathbf{k}) = \frac{1}{\Delta X} \sum_{\ell \in N_p} \Delta x_\ell f(x_\ell) e^{-2\pi i \mathbf{k} \cdot \mathbf{x}_\ell},$$

$$f_k(x_\ell) = \hat{f}(\mathbf{k}) e^{2\pi i \mathbf{k} \cdot \mathbf{x}_\ell}.$$ (15)

The dot denotes the inner product of two vectors; $\Delta x_\ell$ denotes the spatial size associated with the irregular sample, and $\Delta X$ denotes the total summation range in high dimensions. Replacing equation 11 with equation 15, the generalization of the ALFT to multidimensions is straightforward. The subtraction becomes

$$f^\alpha(x_\ell) = f(x_\ell) - f_k(x_\ell).$$ (16)

**SYNTHETIC EXAMPLE**

Although the problem addressed is regularization of 3D seismic data, the discussions and demonstrations have been focused on a 1D signal. Here, we show a simple test for seismic trace interpolation. In Figure 10, there are four seismic events with different amplitudes and dips. The seismic wavelet is a Ricker wavelet with a peak frequency of 20 Hz. Two of the events are aliased. Figure 10a is the ideal sampling with trace spacing of 25 m. In Figure 10b, the traces on even-numbered locations were shifted to the left by 15 m. Plotted on the horizontal axis is the bin number. In Figure 10b, we see the effects of irregular sampling on seismic events; regularization methods are required to interpolate the even traces to the bin center. After applying our ALFT, the interpolation result is shown in Figure 11b. Both the strong events and weak events are accurately reconstructed. For further comparison, the difference between the ideal solution and the ALFT is plotted in Figure 11b. Observe that the differences are small. The result of the interpolation is accurate and is acceptable for real data applications.

**REAL DATA APPLICATIONS**

In general, proper regularization using transform methods involves multidimensional transforms of four spatial coordinates: $x$ and $y$ coordinates of shots and receivers or, equivalently, midpoint $x$, midpoint $y$, offset $x$, and offset $y$. For marine data acquisition, the inline direction is normally well sampled, and the regularization of 3D data can be divided
into two steps. In the first step, the regularization of irregularities of the traces in the inline direction (midpoint x) can be done by a simple sinc interpolation (or truncated sinc interpolation) (Koek and Ongkiehong, 1997; Schonewille, 2000). The regularization along other directions or dimensions (e.g., midpoint y, offset x, and offset y) must be done simultaneously. In the crossline direction (midpoint y and offset y), the data are generally aliased in the cases of complex geologic structures. Thus, regularization along these two dimensions is more difficult because one must deal with both spectral leakage and aliasing.

Our first real data example is marine data from Alaminos Canyon in the Gulf of Mexico, where the spatial sampling in the inline direction (along the midpoint x coordinate) is 12.5 m and the midpoint y sampling is 40–50 m (it is normal for spatial sampling along midpoint y to be about four times larger than that of midpoint x). Dipping events are then well sampled in the inline direction and poorly sampled in the crossline direction. As a result, spatial aliasing occurs at a much lower frequency in the crossline direction. Along the midpoint y direction, there are very few traces at bin center locations. Therefore, data regularization is required to calculate/interpolate all the traces to bin centers.

Figure 12 shows a crossline common-offset section after binning. The complex features are clearly observed (e.g., the salt top reflections/diffractions, crossing seismic events, and amplitude changes along seismic events). Figure 13 shows the data regularization result produced by ALFT. Here, all traces (including the empty bins) have been reconstructed. Comparing this result to the input data (Figure 12), it is clear that the continuity of seismic reflections is much improved.

For a detailed comparison of the input and output, we zoom into the most complex area in the section, which is amplified and plotted as wiggle traces. Figure 14 is the input data in that area. Figure 15 is the regularization result from our ALFT. In Figure 15, the salt-top diffraction event is clearly improved. The blurring effects of binning are removed/attenuated, and the resolution of seismic events is greatly improved.

The output here demonstrates that the algorithm is robust and effective for these complex seismic data although small artifacts can hardly be observed above the water-bottom reflection. However, they do have small values compared to the seismic events.

In order to show the importance of seismic data regularization for prestack depth migration, Figure 16 shows the prestack depth Kirchhoff migration result where the input is not regularized, and Figure 17 shows the same migration with the regularized data as input. Note that with data regularization, we obtain a better cancellation of migration noise and an improvement in the continuity of seismic reflection data.
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DISCUSSION AND CONCLUSIONS

Our ALFT regularization method is not as efficient as the FFT because of the subtraction. However, we can achieve a relatively efficient approach by using a local sliding window in the application. Furthermore, the ALFT suffers much less from boundary effects than does traditional Fourier regularization. ALFT has proved to be a practical approach to seismic data regularization.

Subtraction plays a key role in the calculation of Fourier coefficients with leakage attenuation. The sequence of solving for the Fourier coefficients from maximum to minimum energy is crucial in the algorithm. Both synthetic and real data applications show that our ALFT method (1) works well on complex geologic seismic data, (2) suffers much less from irregular spatial sampling of data and from boundary effects than other Fourier regularization, (3) is robust and stable with noisy data, and (4) is not as efficient as FFT (the efficiency comes from a sliding window technique).

The field data experiments also show the capabilities of bin centering and interpolation into small holes (less than 200 m). The method is not expected to solve all seismic data reconstruction problems, and it may fail to work when the input data has severe aliasing. The generalization of ALFT to higher dimensions is expected to do a better job.


